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COMMENT

Comment on 'Relativistic extension of shape-invariant potentials'

Antonio S de Castro

UNESP Campus de Guaratinguetá DFQ, Caixa Postal 205, 12516-410 Guaratinguetá SP, Brazil

E-mail: castro@feg.unesp.br

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Abstract

This comment criticizes the recently published approach of Alhaidari for solving relativistic problems. It is shown that his gauge considerations are inaccurate and that the class of exactly solvable relativistic problems is not as large as the author claims.

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In a recent paper [1], Alhaidari presented solutions for a number of relativistic problems. This paper followed [2], in which the author presented solutions for the Dirac–Coulomb, Dirac–Morse and Dirac–Oscillator problems. Alhaidari started from a static spherically symmetric electromagnetic field and chose a gauge such that the space component of the vector potential is given by $W(r)\hat{r}$ and he sought 'an alternative and proper gauge-fixing condition'. Alhaidari's letter [2] has been criticized by Vaidya and Rodrigues [3]. In particular, the radial Dirac equation in Vaidya and Rodrigues's comment is in contradiction with that found by Alhaidari. Part of the inconsistency among those equations might be elucidated by noting that the authors use different forms for the Dirac spinors. There is a factor i multiplying either the upper or the lower components of the spinor in order to make the radial functions real for bound-state solutions. Alhaidari multiplied the lower component by the factor i whereas Vaidya and Rodrigues multiplied by the upper component. Nevertheless, the most serious source of contradiction arises due to an error in Alhaidari's radial equation. This error can be seen by noting that $W(r)\hat{r}$ behaves in the same way as the momentum $\overrightarrow{p}$ operator under the change $\overrightarrow{r} \to -\overrightarrow{r}$ so that $W(r)$ should appear in the same way as $\mathrm{d}/\mathrm{d}r$, namely $-W(r) + \mathrm{d}/\mathrm{d}r$ in the second line of the matrix equation (1). The space component of a vector behaves in the same way as a pseudoscalar under the space reflection transformation, however $W(r)$ should not behave as the term κ/r inasmuch as it is not a pseudoscalar potential. In fact, the space component of a vector can always be gauged away, in the relativistic as well as in the nonrelativistic wave equations, and the wavefunctions with and without the field just differ by a phase factor. Needless to say, these considerations are sufficient to invalidate the gauge considerations of Alhaidari's approach.

Notwithstanding, Alhaidari's strategy for transforming the Dirac equation into a Schrödinger-like one is effective solving the Dirac–Oscillator (with a well known pseudoscalar Lorentz structure) and the 'Dirac–Rosen–Morse I' potential with an appropriate mixing of pseudoscalar and vector Lorentz structures.

The spin–orbit coupling parameter κ is defined as

$$\kappa = \begin{cases} -(j+1/2) = -(l+1) & j = l + 1/2 \\ +(j+1/2) = l \qquad (l \neq 0) & j = l - 1/2 \end{cases}$$

so that $|\kappa| = 1, 2, 3, \ldots$. Thus, Alhaidari's strategy is also effective in solving the Dirac–Coulomb problem only if $\gamma = \sqrt{\kappa^2 - \alpha^2 Z^2}$ can be expressed as $l\,(l+1)$; this is so because the centrifugal barrier in the Schrödinger-like equation has the characteristic term $\kappa(\kappa+1)$. On the other hand, all the relativistic potentials with $V = 0$ presented in [1], such as the Dirac–Rosen–Morse II, Dirac–Scarf and Dirac–Pöschl–Teller potentials, have the centrifugal barrier with the factor $\kappa(\kappa+1)$ in the corresponding Schrödinger-like equations. We believe the author has misunderstood the implication of restricting himself to S-wave solutions ($l = 0$) thinking that he could eliminate the centrifugal barrier. However, any integer value of the parameter κ is permissible except $\kappa = 0$.

To summarize, putting aside the question about gauge invariance, Alhaidari's strategy for transforming the Dirac equation into a Schrödinger-like one does not enlarge the class of exactly solvable potentials in the Dirac equation as much as it might appear at first sight.

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